Introduction

This unit focuses on using patterns to investigate arithmetic and geometric sequences. We will be able to use rules to determine any term in the sequence, or the sum of n terms in the corresponding series. We will also use rules to determine the sum of infinite geometric series, and solve problems using sequences and series.

Card Tower

A card tower has three rows:

The top row (Row 1) has three cards.

The second row (Row 2) has six cards.

Continue drawing the pattern for three more rows, and fill in the information below:

Table A:

Row Number	1	2	3	4	5
Number of Additional Cards	3	6			

Table B:

Row Number	1	2	3	4	5
Number of Triangles in the Row	1	3			

Pascal's Triangle

Pascal's Triangle (named after the French mathematician Blaise Pascal) has rows that begin and end with the number 1.

Every other number is the sum of the numbers above it.

Continue the pattern for three more rows, and fill in the tables:

Table C:

Row Number	1	2	3	4	5	6	7
Sum of the Numbers in the Row	1	2					

Triangles...So Many Triangles...

An equilateral triangle has sides of length 64 cm. A smaller triangle is placed inside the first by joining its vertices to the midpoints of the first triangle.

Continue the pattern for two more triangles:

Complete the following tables:

Table D:

Triangle Number	1	2	3	4	5
Length of Side	64	32			

Table E:

Diagram Number	1	2	3	4	5
Number of Triangles	1	3			

Hot, hot, hot

The average daily temperature in Myrnam on June 25 was 8°C. For the next ten days, the temperature decreased by 4°C each day. Complete the following table:

Table F:

Day Number	1	2	3	4	5
Temperature (°C)	8				

Puttin' it Together

In each of the tables above, the top row is made up of the **natural numbers** 1, 2, 3, 4, 5, etc., and the bottom row is a **sequence** of related numbers in a specific order.

The first table gives us the sequence $3, 6, 9, 12, 15$. The individual items in a sequence are called **terms**.

The first term, t_1, is equal to 3. The second term, t_2, is equal to 6; $t_3 = 9$, etc.

Exercises

Complete each table below with the information you gathered in the first part of the notes. Then, **write a description of how the sequence changes** and **determine the next two terms** of the sequence.

Table A:

n	1	2	3	4	5
t_n	3	6			

The next term can be calculated by:

$t_6 =$

$t_7 =$

Table B:

n	1	2	3	4	5
t_n	1	3			

The next term can be calculated by:

$t_6 =$

$t_7 =$

Table C:

n	1	2	3	4	5	6	7
t_n	1	2					

The next term can be calculated by:

$t_8 =$

$t_9 =$

Table D:

n	1	2	3	4	5
t_n	8				

The next term can be calculated by:

$t_6 =$

$t_7 =$

Table E:

n	1	2	3	4	5
t_n	1	3			

The next term can be calculated by:

$t_6 =$

$t_7 =$

Table F:

n	1	2	3	4	5
t_n	1	3			

The next term can be calculated by:

$t_6 =$

$t_7 =$

Different Types of Sequences

We will (mostly) study two different types of sequences in this course.

When we add a constant (either positive or negative) to a term to determine the following term, this is called an **arithmetic sequence**.

When we multiply a term by a constant (either positive or negative) to determine the following term, this is called a **geometric sequence**.

There are other possible types of sequences - for example, the **Fibonacci Sequence** is defined by letting the first two terms equal 1, and then each successive term is the sum of the two terms that precede it.

Classify the sequences in Tables A - F as either arithmetic or geometric:

Sequences That Never End...

Finite Sequence: a sequence that ends after a specific number of terms

Infinite Sequence: a sequence that has an unlimited number of terms

Sequences, Functions, and Graphing, Oh My!

We can think of sequences as **functions** that relate the set of natural numbers to the terms of the sequence.

The **domain** of the function is the set of natural numbers, and the **range** of the function is the set of terms of the sequence.

Some sequences can be represented by linear functions, and some sequences can be represented as non-linear functions.

Plot the points (n, t_n) from Tables A - F on a piece of graph paper. What do you notice about each?

Sequences in which the next term is determined by adding a constant to the previous term are called **arithmetic / geometric** sequences. These sequences can be represented by **linear / non-linear** functions.

Sequences in which the next term is determined by multiplying the previous term by a constant are called **arithmetic / geometric** sequences. These sequences can be represented by **linear / non-linear** functions.

Exercises

For each of the following sequences, determine if the sequence is arithmetic, geometric, or neither, and state whether it is a finite or infinite sequence:

1. $1, 2, 3$
2. $-1, 1, -1, ...$
3. $12, 60, 300, ...$
4. $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots, \frac{1}{2187}$
5. $1, 1, 2, 3, ...$
6. $250, 200, 150, ...$

Write the next three terms of each infinite sequence from the question above:

For each of the following sequences: identify the type of sequence, write the next two terms, and describe a rule which can be used to find the next term, using only addition or multiplication.

1. $5, 10, 15, ...$
2. $5, 10, 20, ...$
3. $4050, 1350, 450, ...$
4. $100, 250, 625, ...$
5. $25, 18, 11, ...$
6. $60, 80, \frac{320}{3}, ...$

The reproduction of bees in nature follows a mathematical sequence. Female bees are born of both a mother and a father. A male bee, however, only has a mother.

Draw seven generations of a family tree for a single male bee.

Complete the following table:

Row Number	1	2	3	4	5	6	7
Number of Bees	1	1					

Is the sequence represented in the table arithmetic, geometric, or neither?

How many female bees are in row 7?

How many bees are in the eighth row?